



## ON THE DISTANCE AND DISTANCE SIGNLESS LAPLACIAN EIGENVALUES OF GRAPHS AND THE SMALLEST GERŠGORIN DISC\*

FOUZUL ATIK<sup>†</sup> AND PRATIMA PANIGRAHI<sup>†</sup>

**Abstract.** The *distance matrix* of a simple connected graph  $G$  is  $D(G) = (d_{ij})$ , where  $d_{ij}$  is the distance between the  $i$ th and  $j$ th vertices of  $G$ . The *distance signless Laplacian matrix* of the graph  $G$  is  $D_Q(G) = D(G) + Tr(G)$ , where  $Tr(G)$  is a diagonal matrix whose  $i$ th diagonal entry is the transmission of the vertex  $i$  in  $G$ . In this paper, first, upper and lower bounds for the spectral radius of a nonnegative matrix are constructed. Applying this result, upper and lower bounds for the distance and distance signless Laplacian spectral radius of graphs are given, and the extremal graphs for these bounds are obtained. Also, upper bounds for the modulus of all distance (respectively, distance signless Laplacian) eigenvalues other than the distance (respectively, distance signless Laplacian) spectral radius of graphs are given. These bounds are probably first of their kind as the authors do not find in the literature any bound for these eigenvalues. Finally, for some classes of graphs, it is shown that all distance (respectively, distance signless Laplacian) eigenvalues other than the distance (respectively, distance signless Laplacian) spectral radius lie in the smallest Geršgorin disc of the distance (respectively, distance signless Laplacian) matrix.

**Key words.** Distance matrix, Distance eigenvalue, Distance spectral radius, Distance signless Laplacian matrix, Geršgorin disc.

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Corresponding Author: Fouzul Atik.

<sup>†</sup>Department of Mathematics, Indian Institute of Technology Kharagpur, India (fouzulatik@gmail.com, pratima@maths.iitkgp.ernet.in).