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FROM CONVERGENCE IN MEASURE TO CONVERGENCE OF MATRIX-SEQUENCES THROUGH CONCAVE FUNCTIONS AND SINGULAR VALUES*

GIOVANNI BARBARINO† AND CARLO GARONI‡

Abstract. Sequences of matrices with increasing size naturally arise in several areas of science, such as, for example, the numerical discretization of differential and integral equations. An approximation theory for sequences of this kind has recently been developed, with the aim of providing tools for computing their asymptotic singular value and eigenvalue distributions. The cornerstone of this theory is the notion of approximating classes of sequences (a.c.s.), which is also fundamental to the theory of generalized locally Toeplitz (GLT) sequences, and hence to the spectral analysis of PDE discretization matrices. Drawing inspiration from measure theory, here it is introduced a class of functions which are proved to be complete pseudometrics inducing the a.c.s. convergence. It is also shown that each of these pseudometrics gives rise to a natural isometry between the spaces of GLT sequences and measurable functions. Furthermore, it is highlighted that the a.c.s. convergence is an asymptotic matrix version of the convergence in measure, thus suggesting a way to obtain matrix theory results from measure theory results.

Key words. Singular value and eigenvalue asymptotics, Convergence in measure, Matrix-sequences, PDE discretizations, Generalized locally Toeplitz sequences, Concave functions.

AMS subject classifications. 15A18, 28A20, 15A60, 15B05, 26A51.

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†Faculty of Mathematical and Natural Sciences, Scuola Normale Superiore, Pisa, Italy (giovanni.barbarino@sns.it).

‡Institute of Computational Science, USI University, Lugano, Switzerland (carlo.garoni@usi.ch), and Department of Science and High Technology, University of Insubria, Como, Italy (carlo.garoni@uninsubria.it).