Translation of Function: A Study of Dynamic Mathematical Software and Its Effects on Students Understanding of Translation of Function

Jane Anne Taylor

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Translation of Function:

A Study of Dynamic Mathematical Software and Its Effects on
Students Understanding of Translation of Function

By

Jane Anne Taylor

Plan B Project
Submitted in partial fulfillment of the requirements
for the degree of Masters in Science in Natural Science/Mathematics
in the Science and Mathematics Teaching Center at the University of Wyoming, 2013

Laramie, Wyoming

Masters Committee:

Associate Professor Dr. Lynne Ipina, Chair
Associate Professor Dr. Scott Chamberlin, Co-Chair
Associate Professor Dr. Linda Hutchison
Assistant Professor Dr. Ana Houseal
Abstract

The purpose of the project was to review the concept and history, research, and teaching methods associated with functions and their transformations, develop a unit on transformations of functions using the Dynamic Mathematical Software Geogebra, and make recommendations about some suitable teaching methods for this subject in high school Algebra classes. In order to realize this, relevant literature was reviewed and considered for developing the functions unit. Data from a pre- and post-assessment and a district assessment was analyzed and interpreted to inform future teaching methods. Geogebra activities were used to teach transformations of functions and then evaluated to determine their effectiveness. Conclusions and recommendations for teaching the transformations of functions are offered.
ACKNOWLEDGMENTS

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Chapter One

Introduction

During the summer of 2010, I along with four colleagues attended the First North American Geogebra Conference in Ithaca, New York. Geogebra is free software developed for mathematics teachers to use in conjunction with their curriculum. Conference presenters demonstrated the tools available within Geogebra and shared lesson plans developed by users of the software from around the world. One of the tools of significance to me was the slider tool. The slider tool provides users the opportunity to manipulate specific variables in a function. As a geometric and algebraic parameter, interactive activities have been created with sliders so that students may explore and manipulate mathematical ideas and see the effect on a graph. After teaching the unit on functions and the transformations of function without much success for the last five years, Geogebra seemed like an answer to my frustrations. I created a few lessons using Geogebra to explore the parameters of quadratic, square root, and absolute value functions. Students working within the lessons that I created seemed to grasp the visual aspect of transformations of functions with moderate success. I wondered if this tool might help our struggling staff to teach functions, to help students grasp the function concept. This issue of using Geogebra to facilitate student understanding of functions is one of the many questions I hope to answer with this project.

The question of teaching transformation of functions to gain a deeper level of thinking has been on my mind since before taking a required history of mathematics course. This class introduced many influential mathematicians and the work they are credited with. As we studied each passing period of mathematics, it became apparent that the definition of function was changing to meet the needs of the mathematicians at that time. Studying and understanding the
background of function gave me confidence in the approach I took to explain function to my high school students.

The definition of the function concept has been evolving since Nicole Oresme (1323-1382), one of the earliest mathematicians on record, demonstrated independent and dependent quantities in his geometric theory of latitudes (Ponte, 1992). During the 19th and 20th centuries, the concept of function has changed from a curve described by a motion to expressions using variables and constants to describe a relationship between two or more variables’. In the early 20th century, set theory was developed and introduced as another way to represent functions. Even in today’s mathematical curriculum and textbooks, there are multiple definitions of function. Which definition of function are teachers to use and how should it be taught? Table 1 lists a few definitions found in research articles.

Table 1

<table>
<thead>
<tr>
<th>Author</th>
<th>Definition</th>
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<tr>
<td>Lial and Miller (1972)</td>
<td>A function is a special type of relation in which, for each value of the first component, there is only one value of the second component.</td>
</tr>
<tr>
<td>Hartter (2009)</td>
<td>The idea of a function as a rule that describes a relationship between changes in the independent and dependent variables.</td>
</tr>
<tr>
<td>Charles, et al. (1996)</td>
<td>If two quantities, x and y, are related so that there is only one value of the dependent variable (y) associated with any value of the independent variable (x), they y is a function of x.</td>
</tr>
</tbody>
</table>

For the purpose of this paper, I will be using the following definition: A function is a relation in which for every input, there is exactly one output.

With the introduction of “new math” in the 1950s-60s, the function concept as taught in schools changed. Middle level students were introduced to set theory with the view that numbers
were thought of as a collection of objects, and even very young children practiced drawing unions and intersections of sets. Diagrams like that in Figure 1 were typical, and we began using words like mapping to represent functions. The 1970s brought a “back-to-basics” movement, and the 1980s wanted teachers to teach for understanding.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
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<tbody>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>-17</td>
</tr>
<tr>
<td>-9</td>
<td>11</td>
</tr>
</tbody>
</table>

Rule: Input *2 + 1

*Figure 1. Kidney Bean Mapping.*

**Statement of the Problem**

The National Council of Teachers of Mathematics (NCTM) proposed in the *Curriculum and Evaluation Standards for School Mathematics* that, “one of the central themes of mathematics is the study of patterns and functions” (NCTM, 1989). According to the Common Core State Standards, one of the biggest ideas in any Algebra course taught is functions and how to represent them. Yet many teachers have not been prepared on how to teach the function concept and some teachers still do not have a grasp of the function concept.

With the advancements of educational technology in the classroom, the behavior of functions can be explored in ways other than with paper and pencil. According to Holden, Ozok, and Rada, “teachers are the ‘gatekeepers’ in the success of educational technology” (2008). With the concept of function being largely visual, it makes sense that teachers should be familiar and proficient with the educational technology and tools that will assist this type of presentation and are available for use in the classroom.
Since the mid-1980’s, computer software and graphing calculators have been developed and improved to provide mathematics teachers with tools to use in all areas of their curriculum. A more recent addition to computer-based technology, Dynamic Mathematical Software (DMS), allows students to “explain, explore, and to model mathematical concepts and the relationships between them” (Zengin, Furkan, & Kutluca, 2012, p. 184). Software is also easier for math teachers to access. Dynamic geometry software, as an example, allows users to create and then manipulate geometric figures. Most DMS contain Computer Algebra Systems (CAS) capabilities, which allow the user to operate mathematical expressions in symbolic form. Some examples of DMS are: Geometer’s Sketchpad®, Cabri®, Wolfram Alpha®, Maple®, TI-Nspire® and Geogebra®.

While there has been substantive research on the effects of DMS in many areas of the mathematical curriculum, there is a limited amount of research done on function transformation. The transformation of a function moves the graphic representation of the function left, right, up, down, or reflected over the x- or y-axis depending on where numbers are placed in the expression. Figure 2 displays a parabola function with vertex at (0, 0) moved to (3, 3). This is a transformation of three units to the right and three units up and is also represented in symbolic form.
Before using Geogebra, Geometer’s Sketchpad, or a graphing calculator, students were shown this type of transformation through lecture and notes and were expected to be able to take the transformation and write it in symbolic form (an equation). This instruction was met with little success. After the Geogebra conference, function transformation was taught using a few Geogebra lessons that I created. The lessons had some manipulation and discovery elements to them and the students seemed to understand how transformations worked graphically. There were still some problems taking a graphical representation of a transformation and representing it as an equation in the form \( f(x) = a(x \pm h) \pm k \). This project will focus on using many Geogebra lessons either found at the Geogebra website or created by me, with concentration on the connection between the graphical and algebraic representations.

**Purpose and Research Questions**

The purpose of this project was to determine whether using Geogebra and other educational tools, such as the graphing calculator, will help students to understand the idea of function and how the graphic representation of a function coordinates with, and complements, its
algebraic representation. Another purpose was to determine if students develop a deep understanding of function using DMS that will continue to assist them in the following units.

This investigation provided the researcher with data about how students think and respond to the use of DMS in regards to the concept of functions and their transformations. It also enlightened the researcher about student learning and the strengths and/or weaknesses of the lessons using Geogebra.

The driving force behind the action research and the literature review involved these four skills:

1) Representing patterns with a table, ordered pairs, and mapping.
2) Being able to verbalize a function.
3) Modeling a function graphically.
4) Taking a function that is in graphical form and expressing it in symbolic form.

Two guiding questions addressed while collecting and interpreting the data were:

1) What does it mean for students to understand function transformation?

2) What demonstrated skills show mastery of function transformation?

These questions will be the focus of the rest of this paper.
CHAPTER 2

Review of the Literature

Introduction

The purpose of this literature review is to focus on students’ abilities to transform functions using the Dynamic Mathematical Software (DMS) Geogebra. Since function, as taught K-12 formal education, has a relatively short mathematical history, this study begins with a history of the function concept and the teaching of function. The pedagogy associated with good classroom practice will guide both the design and implementation of this project. Misconceptions and obstacles to learning are of particular interest for this project because of the student-centered design that will be employed in this project. Finally, this chapter concludes with a section summarizing some of the research-based lessons about the use of technology to teach functions and transformations.

The History of Function

In 1989, functions were considered by the NCTM to be the single most important idea in mathematics. One modern definition of function builds off the concept of a relation: “a relationship between two variables, a connection between two patterns and their commonalities” (NCTM PSSM, 2000, p. 38). An example of a relation could be as follows: A teacher lists the heights of all of the students in his class. The pairing of the student and his/her height is a relation. A function is then defined as “a rule that describes the commonalities between two patterns”. This rule connects every input with one output. So, is the relation from the example above a function? To determine this question, every height listed by the teachers should have one and only one student name that corresponds to that height. If there is more than one student with the same height, then the relation is not a function.
**The 17th century.** The history of mathematics indicates that function was used in many ways throughout the centuries. According to Ponte (1992), functions and derivatives are the foundation of analysis. The appearance of functions as an object of study started at the end of the 17th century. Descartes (1596-1650) led the way with infinitesimal calculus where an equation with two variables represented by a curve showed dependence between variable quantities. Liebniz (1646-1716) was the first to use the term “function” in 1673, he also coined the terms “constant”, “variable” and “parameter” (Ponte, 1992).

**The 18th century.** Throughout the 18th century, the naming of functions with “analytical expressions” would remain unchanged (Ponte, 1992). Kleiner (1989) states, “that the evolution of the function concept can be seen as a tug of war between two elements, two mental images: the geometric and the algebraic” (p. 1). The third element, the “logical” definition, enters later. Euler (1707-1783) wrote the first work, *Introductio*, in which function plays a central role. He “begins by defining a function as an ‘analytic expression’, that is, a ‘formula’” (Kleiner, 1989, p. 3). Euler’s entire approach is algebraic without a single picture or drawing.

**The 19th century.** The 19th century brought about clarifications that changed the nature and meaning of function. There were many debates among Euler, D’Alembert (1717-1783), and Bernoulli (1700-1782) about what defined a function. Many “classes” of function were created and counterexamples played an important role in highlighting relationships, clarifying concepts, and leading to the creation of new mathematics (Kleiner, 1989). Fourier (1768-1830) entered the scene in 1822 when his work on heat flow in material bodies “had a fundamental and far-reaching impact on subsequent developments in mathematics” (Kleiner, 1989, p. 8). Fourier’s work put analytic expressions of a function and geometric representations of a function on equal footing. Prior to the 20th century, mathematicians were used to the fact that a formula such as
\[ y = x^2 + 2x - 3 \] specifies a function that produces a new number \( y \) from any given number \( x \). Then, Dirichlet (1805-1859) proposed forgetting the formula and concentrating on what the function does in terms of input-output behavior. Finally, Cantor (1845-1918) initiated the development of set theory, which led to ideas of relation in the 20th century.

The “new math” of the late 1950’s and 1960’s was led by three causes: the amazing advances made in mathematical research; the automation revolution that led to the construction of machines with enormous capabilities; and the development of computers that were able to perform computations more quickly and efficiently (Glennon, 1973). Another more visible reason for the development of “new math” was Russia’s first effort in space, Sputnik in 1957. According to The School Mathematics Study Group (SMSG), established in March, 1958, “it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand” (Begle, 1968, p. 19). The SMSG called for three foci: “an improved curriculum offering a deeper understanding of the basic concepts and structure of mathematics; the attracting and training of more of those students capable of studying mathematics with profit; and providing all help possible for those preparing to teach the new courses” (Begle, 1968, p. 19). American schools began to use new textbooks based on the set-theory that Cantor initiated in his research. For example, the process of solving an algebraic equation required an explanation as to why each step was used to isolate the variable. To develop the concept of number, non-standard numeral systems (such as binary) were used in problems. Teachers used the function concept as a common theme in this new material.

Some mathematics educators may believe that high school and college students have trouble with the function concept because it is introduced in such an abrupt and abstract way.
However, the goals for including function in the math curriculum are to make functions an integral part of mathematics, make functions a unifying concept across the entire curriculum, use functions to help with mathematical thinking and reasoning, and relate functions to real life situation (The Place of Functions). The term function has many different meanings besides its use in mathematics. Function has been defined as purpose, event, role, and mechanism. Students need to understand what term teachers are referring to and develop a mental image for each.

**Introducing Functions in K-6th Grades**

Willoughby (1997) believes that if functions are introduced in a concrete way and then gradually abstracted over a lengthy period of time, the students will have a better concept of function. “This understanding is demonstrated by their ability to solve problems involving simple function rules; by the ease with which they understand and use variables; by their ability to make and interpret graphs of functions; and by their understanding of real-world examples of functions” (p. 197). Willoughby describes an activity that he and other teachers used in Kindergarten through 6th grade to develop the concept of function. Starting in Kindergarten, students are introduced to the idea of function by using a box to hide a student. Other students come up to the box, insert an object, and the box spits out more or less of the same object. Students then determine what the “rule” of the box is. This concept continues through 5th and 6th grades, only with more advanced ideas. Willoughby also states that, “the important and general point here is that if a concept is approached slowly and through the learners’ experience and if abstract symbolism is delayed until understanding has occurred, most subjects can be learned and understood by children” (p. 199). The purpose of symbolism is to reduce necessary work, and if symbolism is introduced before the concept is clear, problems understanding function can occur.
Developing Algebraic Reasoning and Thinking

In 2000, NCTM recommended that middle school algebra teachers to “assist students’ transition to formal algebra by developing meaning for the algebraic symbols that students use” (Lannin, 2003, p. 342). Lannin (2003) explains that teachers could begin to address the NCTM’s recommendations “by helping students move from specific numeric situations to develop general rules that model all situations of that type” (p. 342). According to Kriegler (2008), there are two major elements of algebraic thinking: the development of mathematical thinking tools and the study of fundamental algebraic ideas. Mathematical thinking tools include problem solving skills, reasoning, and representation skills. Algebraic ideas include algebra as an abstract arithmetic, as a language and as a tool to study functions and modeling. When students understand and connect these algebra skills in consistent ways, mathematical proficiency is likely to increase.

An example of a lesson to model algebraic thinking by Lannin (2003) introduces a cube sticker problem that allows students to make generalizations using patterns. The five strategies used to solve the sticker problem in Figure 3 are counting, recursion (a repeated application of a process), whole-object, contextual, guess and check, and rate-adjust. If students do not have a strong understanding of the connection between addition and multiplication, they may have a hard time moving past the use of the recursive strategy (Lannin, 2003). Lannin continues, stating that, “one of the advantages of generalizing numeric situations is that the variables actually represent varying quantities” (2003, p. 347). This article parallels Willoughby (1997) in that students need to have situations that make sense to them so that they can work with concrete ideas. Once these concrete ideas are commonplace and easy to recognize, abstract symbolism can then be used to generalize the situation to more complex problems.
A company makes colored rods by joining cubes in a row and using a sticker machine to place “smiley” stickers on the rods. The machine places exactly 1 sticker on each exposed face of each cube. Every exposed face of each cube has to have a sticker; this rod of length 2, then, would need 10 stickers.

1. How many stickers would you need for rods of lengths 1–10? Explain how you determined these values.
2. How many stickers would you need for a rod of length 20? Of length 50? Explain how you determined these values.
3. How many stickers would you need for a rod of length 157? Of length 213? Explain how you determined these values.
4. Write a rule that would allow you to find the number of stickers needed for a rod of any length. Explain your rule.

Figure 3. Sticker Problem.

Curriculum Design and Learning

Some authors have claimed that the function concept is the “fundamental concept of algebra” (Hartter, 2009, p. 201). A conceptual understanding of function includes connections between multiple representations such as graphical, verbal, numerical, and analytical. A concept image is defined as a student’s experience with examples and nonexamples of a concept. According to Hartter, (2009) carefully designed classroom activities can enhance a student’s mental picture of function. Due to the lack of conceptual understanding of function, Hartter designed an activity that addressed the verbal representation of functional relationships. “The idea of a function as a rule that describes a relationship between changes in the independent and
dependent variables is essential to students’ conceptual understanding” (p. 202). The ability to identify patterns and organize the data to represent situations is critical to algebraic thinking.

The following problem is taken from Hartter’s (2009) activity: Determine whether the following relationships could be descriptions of functions. For each relationship that you decide is a function, state its input and output variables. If the relationship is not a function, explain why not.

- Number one: the volume of a cube varies as the length of one if its edges are changed.
- Number two: To each student enrolled in the course, a grade (A-F) is assigned.

The answer to number one is yes, and to number two could be yes or no depending on how the student answers. If students are listed as inputs, then each student has exactly one output, their grade. If grades A-F are the inputs, then each grade could have more than one student’s name.

These examples of descriptions of functions provided rich discussion to help attain a more robust concept image of function. After Hartter (2009) uses her activity with her students, she finds that students need a lot of practice with verbal descriptions of functions. According to Vinner and Dreyfus (in Hartter, 2009), even if students can give a correct verbal description, they do not necessarily have a consistent concept image with that definition. Hartter states that in accordance with NCTM’s Principles and Standards for School Mathematics (2000) she wants students to be able to understand function by using various representations for them and converting between these representations. It is up to teachers to provide a variety of activities to help students achieve a strong concept image of function.

Being able to make connections among multiple representations is an important communication tool. “The ability to create, interpret, and translate among representations gives students powerful tools for mathematical thinking” (Kriegler, 2008, p.4). Kriegler introduces a
garden problem as an example that leads to number patterns and algebraic symbols. This problem can be approached in multiple ways and watching students develop strategies to solve this problem can be exciting. Some ways that this problem has been solved is by building or drawing gardens and counting the tiles (modeling), creating a table that keeps track of length and number of tiles (numerical representation), identifying patterns within the numbers and logical guesses about extending the problem can occur (inductive reasoning), and verbal or a symbolic rule might be created and tested (representation). Kriegler (2008) believes that,

…algebra is often viewed as a tool to study functions and mathematical modeling.

Seeking, expressing, and generalizing patterns and rules in real world contexts; representing mathematical ideas using equations, tables and graphs; working with input and output patterns; and developing coordinate graphing skills are mathematical processes and procedures that build algebraic skills (p. 6).

The garden problem is a great activity to introduce patterns that lead to multiple representations of the same function.
The Garden Problem

Gardens are framed with a single row of tiles as illustrated here. (A garden of length 3 requires 12 border tiles.)

A. How many border tiles are required for a garden of length 12?
B. How many border tiles are required for a garden of length “x”?
C. How long is the garden if 152 tiles are used for the border?

Figure 4. The Garden Problem.

A study by Akkoc and Tall (2002) focused on “the concept definition and its interpretation in a variety of representations of function” in the Turkish math curriculum (p.2). Students were introduced to function as stated in the next paragraph. The purpose of the study was to explore the curriculum for function and the problems it presents for students, especially with the use of the formal definition and concept images that occur. One hundred students from four different secondary schools in Turkey were given a questionnaire in which they were asked if different graphs, equations, set diagrams, sets of ordered pairs were function or not (Akkoc & Tall, p. 3). It was found that when looking at set diagrams and ordered pairs, students refer to the everyday definition of function to determine if it was a function or not. When graphs or formulas are given to determine if an equation is a function or not, students rely on exemplars or models. The authors’ analysis of function seems to suggest that different representations are given and interpreted in slightly different ways.

Another conducted by Akkoc and Tall (2005) looked at the curriculum design of function and students’ understanding. They based their study on a claim by Pat Thompson; “if students do
not realize that something remains the same as they move among different representations then they see each representation as a ‘topic’ to be learned in isolation” (2005, p.1). The study was conducted in Britain and Turkey in diverse high schools (public, private, and selective) with students in 10th-12th grade. In Britain, curriculum is based on students remembering their experiences instead of focusing on the definition of function. In the Turkish curriculum, function is introduced using a “formal” definition and then given as a visual representation. Akkoc and Tall (2005) explain that “after this introduction, various examples of functions are given in different representational forms, such as sets of ordered pairs, set correspondence diagrams, tables, graphs, and formulas” (p. 2). The results of the study indicate that, “very few students could coherently focus on the definitional properties of the function concept for various aspects of functions” (p. 7). There was a disconnect between the students’ understanding of function and the curriculum design. Students tended to concentrate on the individual properties of each function representation without connecting them together.

During an annual Seminar at the Park City Mathematics Institute, a presenter shared a paper entitled The Place of Functions in the School Math Curriculum. This paper suggested that there are three ways to introduce function to students: (a) provide experiences with classes of functions; (b) provide a general definition, and (c) provide students with experiences in representing various classes of functions (The Place of Functions). It is noted though that some pitfalls to this method of introduction could limit the concept image for functions, the definition does not have much meaning for them, and students have little opportunity to see the general concept of functions.

The author from this seminar believes that students need to be introduced to the various notations of function as well as the various representations. To develop a rich concept image,
students should encounter the following representations: symbolic, graphic, diagrammatic, verbal, tabular, and implicit. The teacher is in charge of students developing a concept image that matches a complete definition of functions. Students are not making the connections among the different representations and need to explore and develop deep and rich understanding of function. The author also believes that “good teachers will provide rich environments, emphasizing making connections among different representations, and allow students to take the time to develop such” (The Place of Functions, 2009, p.4). One way that is mentioned to motivate students towards this end is to provide technology and multiple opportunities to support students learning of functions. Some benefits mentioned are the use of multiple representations, modeling, explorations of properties, consolidating function as object, manipulative tools to avoid tedious manipulations, and environments for exploring other kinds of functions. Pitfalls were also given when using technology. Examples of such pitfalls are: (a) concept images being restricted by technology; (b) a balance of human activity and understanding; and (c) use of computers with the caveat that sufficient time is necessary for students to learn software tools (The Place of Function, 2009).

**Obstacles in Student Learning of Function Transformations**

Translation ability “refers to the psychological processes involved in going from one mode of representation to another, for example from an equation to a graph” (Gagatsis & Shiakalli, 2004, p. 645). One of the main goals in teaching functions is for students to be able to change from one representation to another. Extensive research shows that if students have translation ability they will be better problem solvers than those who do not have translation ability. According to Gagatsis and Shiakalli (2004), “A translation involves two modes of representation – the source (initial representation) and the target (final representation)” (p. 646).
All across the mathematics teaching community, it is widely agreed that representations are an inherent part of the concept of function and that functions are an important piece to each student’s math education. Gagatsis and Shiakalli (2004) believe that math students and teachers tend to avoid the graphical representation of function and prefer the algebraic expression.

Multiple studies have been conducted to determine students’ way of thinking when learning function transformations. Some studies have shown that students tend to see line segments moving as horizontal translations instead of vertical. Both of the translations may be associated with correct strategies, but the vertical translation strategy tends to be more straightforward when focusing on the equation and the y-axis (Chiu, Kessel, Moschkovic, & Munoz-Nunez, 2001). Chiu, et al. (2001) wrote a case study about an eighth grade student, Paul, who had no prior exposure to function. The goal was to determine how he saw a change in b in a linear function. The study involved 6 hours of videotape involving Paul’s work through linear functions and their graphs. Paul had access to software called GRAPHER®, which allowed him to manipulate algebraic and graphical representations of functions. At first, the student viewed parallel lines as horizontal instead of vertical translations of each other. In other words, a horizontal translation would move a parallel line to the right or left whereas a vertical translation would move the line up or down. When horizontal counting did not work or make sense, Paul was able to use vertical counting with the prodding of his tutor. Paul was able to refine his conceptions by associating it with other conceptions - a conception is a set of coherent and connected ideas associated with a specific strategy or with several related strategies (Chiu, et al., 2001).

Based on this and other similar research studies, understanding linear functions involves the connections between representations. Conceptual change can be seen in many ways. New
conceptions can be added, be connected in different ways, can change, or the relation among several conceptions can change. “Conceptual change can occur when students relate multiple conceptions through integration or differentiation” (Chiu, et al., 2001, p. 222). This study suggested four ways to facilitate conceptual change: “(a) applying them in different tasks, (b) constructing strategies that are as general as possible, (c) comparing different strategies, and (d) generating criteria for evaluating strategies” (Chiu, et al., 2001, p. 246). According to the authors, it is important for teachers to allow for multiple strategies when learning translations.

Another study focused on two tenth-grade honor students’ responses to translations of quadratic functions in a high school in North Florida. The students’ way of thinking on different tasks provided the researchers information about student learning of graphs. This is an important process on helping reduce levels of abstraction when learning functions. Task–based interviews were conducted at the end of a three week unit on quadratic functions (Harel, Fuller, & Rabin, 2008). The students’ classroom tests, quizzes and questionnaires were analyzed for understanding to find out what obstacles they were encountering while being taught translation, determining, interpreting, and solving quadratic equations and using quadratic models. During the interview process, the researcher found that one student used memorized rules to perform tasks given without really knowing why he was doing it. This was termed instrumental understanding, where rules and algorithms are memorized without knowing the reason why. The other student used similarities between the standard form and vertex form of a quadratic equation to answer the task given. Standard form is the following: \( y = ax^2 + bx + c \). Vertex form is given as: \( y = a(x - h)^2 + k \). This student used the \( b \) and \( c \) in standard form as the vertex of the quadratic equation. Harel, et al. (2008) states that, “abstract algebra requires students to
think the meaning of the situation given in a task rather than recalling a memorized and unconnected procedure” (p. 1058).

Students need to be able to interpret and understand situations. If the students’ strategies are known in advance to a lesson, teachers can help students to clear up these obstacles (Harel, et al., 2008). Non-routine problems or engaging activities that forces students to use different strategies would help them to make connections between different representations of the quadratic function. The authors also indicated that computers and mathematical software could also help to bridge the gap from abstract to concrete and vice versa.

A study done at the University of Cyprus in Greece tested 195 university students on six direct translation tasks from one representation to another (Gagatsis & Shiakalli, 2004). On Test A, students had to move from the verbal representation of function to graphical and algebraic representations. Test B, students were expected to move from the graphical form to verbal and algebraic form. The findings of the study found that students did not realize that verbal and graphical modes represent the same concept. When a graphical representation of function was used, the students tended to score lower than with other representations. The authors of this research did point out that teachers and students in Greece tend to shy away from graphical representations of function and concentrate instead on the algebraic representation. According to Gagatsis and Shiakalli (2004), the findings of the study demonstrate that understanding the concept of translation of function is an “essential” prerequisite for problem solving skills. Once again, instructions of functions should include all modes of representation for students to have a better understanding and success with the function concept.
Using Technology to Teach Function Transformation

When teaching mathematics, words and numbers alone are not enough to facilitate genuine student understanding of functions. Multiple forms of communication, including verbal, numeric and visual, are required to convey evidence about questions of any complexity (Murphy, 2009). The use of visual electronic technology has now become a part of our daily lives. Smart Board®, PowerPoint®, applets, graphing calculators and websites are being used by teachers to get their points across to students. According to Murphy (2009), “Visuals help to engage students, to grab their attention and demonstrate how math is relevant to their lives” (p. 1).

Visual language is a powerful approach to express mathematical concepts. Visual literacy is the tool that allows us to interpret this language. Visual learning involves the set of skills that include observation, recognition, interpretation, perception and communication (Murphy, 2009). To understand abstract math concepts, students need to be able to see how they work. Murphy also states, “It is the process of creating mental images or models of a concept that students are able to make the all-important leap from the concrete to the abstract” (p. 4). Visual images are fundamental when learning mathematical concepts.

Taxonomy of Mathematics Instruction Software

While many authors developed their own tool-based taxonomy for educational technology, they all come down to three types of software: instructional, general purpose, and Computer Algebra Systems (CAS). Instructional software is mainly “didactic”, making the user passive (Guyer, 2008). Included in this software are review and practice, math games, the quiz format, and exercise software (static calculations). The instructional software was used for computational skills that did not lead to a deeper understanding (Bos, 2009).
General-purpose software has a visualizing-purpose that includes CAS, information tools, and concept teaching-purpose use. General software uses common programs with an interactive element to help students explore and solve problems in many different mathematical topics (Kurz, Middleton, & Yanik, 2005). An interactive math object helps in that “what once was abstract becomes discernible, tangible, and real. This indeed is a missing piece in the cognitive picture of a math concept” (Bos, 2009, p. 110).

The final software is Computer Algebra Systems (CAS). CAS software includes specific software, environment software, and communication software. Specific software has students use tools such as sliders and animation to delve into a specific mathematical topic, such as Geogebra or Geometer’s Sketchpad®. Environment software allows students to experience virtual applications by investigating mathematics interactively (Kurz, et al., 2005). Communication software emphasizes cooperative learning and out-of-class learning.

According to Bos (2009), when a teacher uses electronic technology in the classroom, the technology needs to retain math and cognitive fidelity. Cognitive fidelity “enables one to make connections by seeing developing patterns that are only possible in the mind” (Bos, 2009, p. 112). As a slider changes the parameter of a variable in an equation, the corresponding graph reacts accordingly. Mathematical fidelity occurs when a mathematical object is “accurate and mathematically precise” (Bos, 2009, p. 112). For example, when a division problem is truncated because the calculator cannot carry out the decimal approximation is considered a limitation. Technology’s limitations are always going to be of concern for teachers. When using interactive math objects, cognitive and mathematical fidelity are important features as well as understanding which specific way each software can be used to enhance classroom instruction. It is important to know the limitations of graphing calculators when graphing certain functions, such as those
with asymptotes. Likewise, it is important to know limitations of DMS when working with symbolic representations of equations.

**Effects of Technology in the Classroom**

Since electronic technology has realized an increase in use for teaching and learning in the classroom, many studies have been conducted accessing students’ performance. Bakar, Ayub, Luan, and Tarmizi (2002) in Malaysia, and Sheehan and Nillas (2010), in an urban, mixed income community, found that computers and graphing calculators made lessons more interesting and fun, increased student participation and motivation, and led to deeper student understanding. Each study used questionnaires to assess student motivation and perceptions about technology, thus they were both self-report studies.

Souter (2001) compared technology-enhanced instruction with traditional algebra instruction in student achievement, motivation, and attitude towards technology. The study was performed in southwestern Georgia using ninety- two students with 52 attending a technology-enhanced class and 40 in a traditional class. According to the data, student achievement was higher among technology users than non-technology users. Students in technology-enhanced classrooms participated more times in class than those not using technology. In student interviews, the majority of students in the technology group gave positive responses relevant to their attitude towards using technology in algebra. “The implications of my study are that integrating technology into the mathematics classroom can increase student achievement, increase student motivation, and foster positive student attitudes” (Souter, 2001, p. 10).

It was stated in the beginning of the article that the NCTM recommended, “…that technology be used wisely by well-informed teachers to support mathematical understanding” (Souter, 2001, p. 1). This seems to be an overwhelming consensus among educators and
researchers alike. Since technology is an important part of teaching mathematics, teachers are responsible for using best teaching practices with the growing emphasis on technology driven instruction. “One of technology’s strengths lies in its ability to support fluid inquiry where students can try various inputted values to observe patterns encouraging students to conjecture, predict, test, and generalize” (Bos, 2007, p. 352). If technology were used to enhance conceptualization instead of just learning a process, students would be able to directly manipulate mathematical objects using the tools to gain a deeper understanding (Bos, 2007). “Multiple representations are a powerful, visual way of showing patterns, and patterns are the building blocks of mathematics” (p. 353). Further research shows that the use of technology allows students to learn crucial algebra concepts more quickly and in depth than peers that did not use technology. “Since algebra concepts, such as functions, serve as a gatekeeper to high school graduation and college admission, a technology enhanced instructional environment that supports a more comprehensive understanding of algebra and functional relationships is worthy of further study” (Bos, 2007, p. 353).

**Educational Technology and Functions**

For the purposes of this paper, many studies were reviewed to determine if there was educational software that would perform better than others when teaching function transformations. In 2008, Bos analyzed the mathematical and cognitive fidelity of the graphing calculator versus the TI InterActive® software on quadratic functions. This study used a pretest-posttest control-group research design. Participants came from two large metropolitan school districts in Texas and included seven teachers, three from one district and four from another. The 95 students were separated into experimental and control groups. Forty-eight students were in the experimental group and 47 students were in the control group. The experimental group
received instruction on quadratic function in the computer lab using TI-Interactive®. This main objective of utilizing this software was to develop the concept through exploration and problem solving. For eight days, the experimental group spent 55 minutes a day in the computer lab working on six lessons designed by the researcher. The control group received instructions using the same sequence but was instructed using lecture, notes, drill and guided practice (Bos, 2007). This group used graphing calculators to graph quadratic equations and record table values.

The findings indicated that the use graphing calculators provided accurate results but was a static representation; this limited the cognitive fidelity (graphs of functions were not capable of moving from their original point). On the other hand, the InterActive® software put the student in the role of investigator. “The cause and effect of cognitive connections were much clearer when using the InterActive software” (Bos, 2008, p. 4405). Students using the TI Interactive® instructional tools scores were statistically significantly higher than the group who did not use the TI software. The difference between the two technology tools was in cognitive fidelity. The results showed that students who struggle with quadratic functions could benefit from this learning environment (Bos, 2007). The challenge is to find software that has both cognitive and mathematical fidelity.

Hazzan and Goldenberg (1997) conducted a study to determine if using a Dynamic Geometry Environment® (DGE) could enhance the understanding of function. The two ways that DGE was being studied in their research was (a) to determine if DGE would increase students’ ideas of function and (b) if it could help the researchers appreciate students’ understanding of function. The main features of DGE examined were how it allowed students to explore, manipulate, and discuss function.
Three second semester senior undergraduate mathematics were interviewed and observed while working with three figures that the researchers designed: a pair of parallel lines, a parallelogram inscribed in a quadrilateral, and collinearity in a rhombus, kite and circle. The researchers were observing the students’ thought processes while manipulating each figure. Prior research showed that students do an adequate job with numerical evaluation but have a hard time analyzing and describing function behavior (Hazzan & Goldenberg, 1997). “This difficulty seems to fade when functions are represented dynamically…especially when numerical information is suppressed, the most salient remaining feature of the display is the functions’ behavior” (p. 274).

The findings of the research showed that when students used DGE as a tool to explore and discuss the concept of function their achievement is statistically higher than those who do not used DGE. This tool allowed students to drag points on the figures and make discoveries for themselves. Even though this research was conducted in 1997, it is still pertinent. The students became active participants in the learning process. Because of this importance of technology use in the classroom, teachers are responsible for finding technology that creates the best learning environment.

**Assets and Pitfalls of Educational Technology**

There is a major worldwide trend in using technology in the mathematical classroom. Studies have been conducted on classroom practices and the effect of technology on students’ understanding of function. While there are positive aspects of using technology, some negatives have also been found. One of the assets of using technology is that it can help with issues of visualization. For example, a problem where the student is trying to find the maximum area of a rectangle inscribed in a right triangle is introduced. This technology could assist with
visualization of the task. “Observing such a transformation…can be a critical step in the development of the function concept” (Assets and Pitfalls, p. 2). The technology also allows the learner to experiment with the different variables of the problem leading to a dynamic experience.

In addition, this dynamic geometry environment can be used by students or teachers to explore and model functional relationships.

The developers of these software tools provided an unusually rich environment in which students may expand their notions of function and may develop and explore a variety of related ideas. The many differences between algebraic (or graphical or tabular) and dynamic-geometric representations of function provide opportunities for alternative experiences that can, with appropriate reflection, greatly expand students’ ideas of function (Assets and Pitfalls, p.3).

To strengthen the concept of function, students can explore the role of parameters and work with families of functions. Using dynamic software can help students get a complete picture of the problem and gain insight regarding how the function ties to graphs and models used.

There were also a few pitfalls mentioned when using dynamic software. Some mathematicians believe there is too much focus on graphical representations of function, that calculators are being used in inappropriate ways, and technical issues that may be involved (Assets and Pitfalls, 2009). Skemp (2006) also fears that technology may push instrumental understanding instead of relational understanding.

Geogebra in the Classroom

According to Edgar Dale’s Cone of Experience, “we remember 30% of what we hear but remember 80% of what we see, hear and utter”. Using educational technology allows the student
to become an active participant. With so many choices, it is often hard for teachers to determine which tool will fit both teaching methods and the students’ needs. “Technological environments may allow teachers to adapt their instruction and teaching methods more effectively to their students’ needs, (NCTM, 2000, p. 11). The process of integrating technology into the mathematics classroom has been slower than the implementation of technology in the schools. Teachers are often hindered by lack of access, basic skills, not being comfortable using computers, and knowledge about effective integration (Hohenwarter, Hohenwarter, & Lavicza, 2008). The NCTM (2005) states that effective use of technology in the classroom depends on teachers. In addition to the typical obstacles to teaching with computers, research suggests that training and support are needed to develop successful technology assisted teaching.

According to a presentation by Arnold (2009), “The primary use of mathematics technology has been distributed between the twin purposes of manipulation and representation” (p. 2). The purpose of Arnold’s study was to find a tool that combines CAS and dynamic geometry software without one aspect dominating the other. CAS is able to manipulate symbolic objects whereas dynamic geometry is a tool with the ability to construct and manipulate geometric objects, which are dynamic and interactive. Arnold (2009) states, “Models may be built geometrically to provide live illustration of algebraic relationships” (p. 3).

Arnold compared three educational software programs: Geogebra®, Geometry Expressions® and TI-Nspire®. He stated, “Geogebra is a great example of an algebraic/graphical conversation but one which in reality offers limited dialogue between the algebraic and geometric” (Arnold, 2009, p. 5). He further explained that the CAS capabilities in Geogebra are one-way and do not create an equal conversation. When used in tandem, Geometry Expressions software and TI-Nspire CAS seem to be the best solution for integrating CAS and dynamic
geometry capabilities. For the purpose of this paper, I plan to use Geogebra since I have been trained in implementing this software in my classroom. I do not have any experience with the other software programs.

Geogebra is a platform-independent free-ware found on the website www.geogebra.org. Any constructions can be exported and saved as a webpage and it becomes a dynamic worksheet for teachers to use in the classroom. Hohenwarter (2007) describes his software, Geogebra, as a link between geometry and algebra that provides a versatile tool for visualizing mathematical ideas. According to Hohenwarter, “CAS is described as focusing on the manipulation of symbolic expressions and DGS concentrates on relationships between points, lines, circles, and so on” (p. 126). The basic objects in Geogebra are described as not only points, vectors, segments, polygons, straight lines, and conic sections but also functions in x. Hohenwarter (2007) states that Geogebra “could be a powerful tool in mathematics education, but to have an impact there are implications for the pre-service educations, and in-service professional development, of teachers of mathematics” (p. 130).

**Tools within Geogebra.** Geogebra supports functions of the form f(x) as self-contained objects. With this support, we can use these graphs for dynamic constructions. While a point is moved along a function, Geogebra updates the coordinate dynamically.

Since the coordinates of the point represent one pair of numbers from the function’s table of values, this figure is a dynamic table of values: by moving the point along the graph we can dynamically investigate any pair of x and f(x) (Hohenwarter, 2006, p. 2). Sometimes presenting the whole graph of a function at once can be a disadvantage. Seeing how a graph is created as the trace of a point (x, f(x)) is moved with a mouse can be advantageous to a student’s critical thinking (Hohenwarter, 2006). A dynagraph representation displays two
parallel lines with x and its function value f(x) is displayed. This representation draws attention to what happens with f(x) as x changes.

Computers make it easy to vary parameters and observe the resulting change to the function’s graph (Hohenwarter, 2006). Geogebra supports the use of parameters by providing a slider tool. The slider tool is commonly used to investigate the parameters in a quadratic function to find a connection among the parameters and the roots, vertex, and transformation of the function. Sliders create interactive activities that encourage students to explore and manipulate mathematical ideas. They are visualizations of variables that can be used in interactive and dynamic constructions and provide dynamic mathematics learning environments. As Bu and Haciomeroglu (2010) state, “In a dragging mode, sliders invite students to manipulate the initial conditions of a problem situation, reflect on their actions, and further identify invariance in a variety of related multiple representations” (p. 213). Sliders can serve various functions for teaching of mathematics, providing students the opportunity to manipulate specific variables. In Geogebra, a slider has two representations: (a) algebraically (variable that has a default for its values); and (b) graphically (a segment that allows the user to adjust the variable through dragging) (Bu & Haciomeroglu, 2010).

From a pedagogical perspective, sliders can be used for constants, independent variables, geometric parameters, and algebraic parameters. For example, in the linear equation y = mx + b, m and b can be represented as sliders to that students can see the effect of m and b on the graph of a line. In another example students can explore a real-world situation in which the independent variable can be represented as a slider to see the effect on the graph. The trace feature can be used to mimic the typically plotted paper-and-pencil method. Furthermore, Bu and Haciomeroglu (2010) intonate, “New interactive technologies provide pedagogically
versatile tools for the teaching and learning of mathematics in a dynamic environment such as Geogebra” (p. 215).

**Research using Geogebra in the Classroom**

Zengin, Furkan, and Kutluca (2012) conducted a study using two comparable math classes in Turkey. The treatment group (n=25) were taught using Geogebra, while the companion group (n=26) were taught with traditional methods. While the students’ pre-test scores on a ten-item assessment were approximately the same, the post-test scores were significantly different.

Reis and Ozdemir (2010) identify parabolas and its subsets analyze whether using Geogebra to teach parabola had an effect on twelfth grade student’s attitude and success. The study was conducted in a private school in Istanbul, Turkey. Five experimental and five comparison groups (n=204) were used for this study. The experimental group used Geogebra and the control group was taught using traditional methods, including lecture (Reis & Ozdemir, 2010). The Geogebra lessons were prepared by the researcher and taken from Geogebra sources. The content of these lessons included the specific pieces of parabolas and its subsets; parabola drawing, identifying elements of parabola drawing, describing a parabola equation, and the max and min values of a parabola (Reis & Ozdemir, 2010). The pre-test shows no statistically significant differences. The differences in post-tests, on the other hand, were found to be statistically significantly different. The experimental group scored higher than the control groups. Reis and Ozdemir (2010) felt that “using visuality helps to engage students, to grab their attention, demonstrate how math is relevant to their lives and explains how mathematical concepts work” (p. 571).
A study of particular interest was conducted by Bakar, et al. (2002). They wanted to determine if there was a difference in performance of students using Geogebra and e-transformation on the topic of transformation. The researchers noted that results from past studies in using computers to help with the instruction of mathematics have been mixed. While some studies have shown that when students use mathematical software, graphing calculators, and Geometers Sketchpad, a positive impact occurred on mathematical thinking, which resulted in students receiving higher scores than peers that did not access such learning tools. On the other hand, other studies have shown that there was no statistical significance between the use of graphing calculators and traditional teaching methods (Bakar, et al., 2002). Another study involving web based interactive tutorials compared to traditional teaching methods resulted in no significant differences on the post-tests. Most of these studies refer to the use of paid software or handheld technologies. There is limited research on the use of open source software.

Bakar et al. (2002) used a quasi-experimental design for this study and randomly assigned students into two groups. One group used Geogebra while the other used e-transformation. There were four phases in this study: (a) the pre-testing phase; (b) introduction to software phase; (c) integrated teaching and learning phase using each software and a learning activity; and (d) the post-test phase. Group one had 40 students while group two had 30. The findings indicated that students who used the Geogebra software and e-transformation showed improvement when comparing the results of the pre and post-test scores of both groups (Bakar, et al., 2002). The study also showed that there was an improvement in student achievement for students who used open source (i.e., free) software as well as for those that utilize paid software. According to this study, open source software works just as well as paid software and should be looked to as an alternative to paid software.
In technology environments, new learning opportunities are provided that can foster visualization and exploration of mathematical concepts. As Hohenwarter, et al. (2008) state, “Students can develop and demonstrate deeper understanding of mathematical concepts and are able to investigate more advanced mathematical contents in ‘traditional’ teaching environments” (p. 136). Their study also aimed to identify effective approaches for introducing Geogebra to secondary school mathematics teachers. It has been shown that teachers who feel comfortable with a software tool are more likely to integrate the tool into their teaching practices. Though there is limited research on the use of Geogebra in the classroom, research on other DGS packages suggest that this software can be effective in the classroom.

Hohenwarter et al. (2008) aimed to “identify difficulties and impediments that participants face during Geogebra workshops and to assess the usability of the software itself” (p. 138). Their three main objectives in this study were to: a) assess the usability of Geogebra and find challenging features and tools that may be difficult to understand; b) establish difficulty levels of the tools to better accommodate needs of future novice users; and c) provide a basis for improvement of introductory Geogebra materials. The study was conducted during a teacher professional development summer institute of the Math and Science Partnership between the Department of Mathematical Sciences at Florida Atlantic University and the school board of Broward County funded by the U. S. National Science Foundation. The two-week institute involved 44 teachers in four daily 70-minute Geogebra workshops. The four workshops evaluated were designed to introduce Geogebra to novice users. Topics ranged from geometric constructions, through linear equations, to exploring functions.

The findings from the questionnaires showed that participants were pleased with the usability and versatility of Geogebra, the tools were rated as “easy-to-use”, complex tasks
negatively influenced tool ratings, and participants tended to spend 50% more time on algebraic input tasks (Hohenwarter et al., 2008). This study showed that offering high-quality professional development for teachers is important for successful technology integration. The study immediately resulted in improvement of future workshops.

The literature has provided insight into many aspects of the function concept. The definition and history of function has evolved and adjusted with the needs of mathematicians and educators alike. In turn, instructional methods for teaching function have also been tweaked. One instructional method that has emerged is teaching functions with Dynamic Mathematical Software. The availability and use of computers as well as DMS has become vital teaching instruments in most schools. With all the available lessons that are online, it is up to teachers to decide what will work best in their classroom. Past studies on educational technology, especially Geogebra, will help me to create and implement lessons that aide in students understanding of the transformation of function.
Chapter 3

Methods

Classroom Setting

The school district used for this study is located in a university town in a sparsely populated western state. Most students’ parents are employed in the educational or retail trade. The senior high school has approximately 750 students accommodating tenth through twelfth grade students. The high school population is comprised of a majority of white students (80%), with smaller populations of Hispanic students (13%), and students from other ethnicities including Asian (3%), African-American (2%), and American Indian (1%). Nineteen percent of the students at the high school qualified for free or reduced lunch and 15 % had Individualized Education Plans (IEP). My 5th hour Algebra II class was used to collect data.

The high school is currently in an older building that was built in 1959. Classes are comprised of approximately 20-30 students. On Mondays, Thursdays and Fridays, class periods are 50 minutes in length. On Tuesdays and Wednesdays, classes are 90 minutes in length with half of the classes held on Tuesdays and the other half on Wednesdays. The room I teach in is on the second floor of the building in the north wing. It is a bright class with a lot of natural lighting. Dry erase boards are along the entire north-facing wall with chalkboards along the south facing wall. Windows encompass the west wall and are from the ceiling to three feet above the floor. Five rows of five desks each are situated in the middle of the classroom with my desk in the southeast corner of the room. Throughout the room are a variety of posters with mathematical formulas and representations. The classroom is open and inviting with plenty of room for students to be able to move their desks for group or individual work.
Institutional Review Board (IRB)

In December 2012, an IRB for this study was approved. Following that, consent forms were sent home with students for their parents/guardians to sign. A total of six males and twelve females became participants in the study.

A few minor problems occurred while attaining consent forms for this research project. Two students from the class could not be used for this study. One of the students turned eighteen in October so I chose not to involve him/her in the research due to extra paperwork. The other student was absent more than present and was eventually put on a homebound track. This student’s data would not have been accurate since I was not the person teaching the material.

The unit being researched started during the final two weeks of first semester and ended the first two weeks of second semester. At the beginning of second semester, two students joined my class after transferring from a different Algebra II class. I did not use the data from these two students since they were not a part of the unit from the beginning.

Pre and Post Testing

The first step in this study was to develop a pre- and post-assessment to gather information about students’ current understanding of the concept of function. Research from the literature review regarding pedagogical strategies and obstacles to students learning functions were used to help find and develop the assessment. According to researchers like Hartter (2009), an understanding of function includes connection among graphical, verbal, symbolic, diagrammatic, tabular, and implicit representations. Akkoc, Tall (2002) and Kriegler (2008) all state that in order for students to grasp the function concept, the following forms should be introduced: a verbal representation in formal and everyday language; finding, expressing and
generalizing patterns and rules; organizing patterns with equations, ordered pairs, tables and graphs; using a set diagram; and working with input and output patterns. Based on this research, the pre-assessment was composed of the following question types: (a) completing a pattern, (b) filling in a table from the pattern, (c) graphing the data on a Cartesian graph to observe a pattern, (d) writing a symbolic expression for the pattern, (e) matching a graph to a story, (f) interpreting function notation, and (g) matching a function rule to its table, graph and equation. The pre-assessment had two versions, A and B. An original pre-assessment contained eight questions that resulted in a length of five pages. Creating two versions allowed me to gain more varied information regarding the students’ prior knowledge of function while keeping the instrument short enough to administer in one 50-minute class period. The post-assessment is a combination of both versions of the pre-assessment. By combining both versions of the pre-test, there was twice as much data for me to interpret. Both pre- and post-assessments were evaluated and approved by the IRB. The pre- and post-assessment can be found in Appendix A.

**Curriculum Planning**

As I was planning for the function unit and investigating how to approach the concept to attain optimal success from the students, I chose lesson plans that bridged the gap between the abstract nature of function and its multiple representations. I was hoping to use different types of educational technology that would present the visual aspect of functions for better understanding among my students. Prior to starting the unit, I administered the pre-assessment.

Since Geogebra was going to be the main tool in my teaching of transformations of functions, I developed a comprehensive tutorial on Geogebra. I went through the Geogebra library of tutorials and cut and pasted the concepts that I felt were the most important for the students to learn. The library presentation area was reserved for twenty-four students to use
laptops and desktop computers. My computer was attached to the projector so that students could follow the directions as they worked. At first, I led students through the first few pages of the tutorials highlighting the pieces I felt were important. Students were then given the opportunity to do some self-exploration and create objects on their own. After thirty minutes of student exploration, I brought everyone’s attention back to the presentation screen and wrapped up the tutorial with some final comments and questions. This activity was done during a ninety-minute block. In the past, I have not done this tutorial but rather had students jump right into the Geogebra lesson at the time.

Since research has placed a lot of emphasis on the verbal aspect of function and how it can be seen in the real-world, the next three days of fifty minute class periods focused on using a Calculator-Based Laboratory (CBL) device, 15 second videos of graphing stories, and representing stories with graphs. To introduce graphing stories, I used the website http://graphingstories.com/# and chose nine 15-second videos. Students watched the video once at normal speed and then watched a second time in slow motion. The video showed how to label the graphs, then the students had to interpret the video and how it would look graphed. For example, one video I chose was the height of a person’s waist off the ground as they were going back and forth on a swing. The class had to label the x- and y-axis with appropriate units and then determine the shape of the graph based on the video.

This activity was a great lead in for me when I presented students with stories and asked them to create a graph to represent the story. During this lesson, the terms independent, dependent, domain, and range were introduced and defined in terms of the stories that were being read. After practicing graphing the given stories, I wanted to introduce students to the idea of a function, particularly how distance was a function of time, while still working with graphing
stories. I found a CBL lesson that asked students to mimic graphs given on the TI-83 graphing calculator. The activity that I chose asked students to use the CBL to create a graph that matched the one given on the graphing calculator. Students would have to determine how far from the CBL to start, if they should walk towards or away from the CBL, and the pace at which they should walk. After students matched a few graphs, they then had to write a story to match graphs given in their worksheet packet. The goal of this activity was for students to identify how slope changed depending on their pace and direction of movement and how the distance represented in the graph was dependent on the time. The students were very involved with the activity, as I had hoped.

The following day during a ninety-minute block, I introduced students to function by asking “What in the world is a function and why is it important?” I felt the best way to answer this question was to pose a problem that involved a linear relationship since the students had worked the entire first semester on this concept. The linear relationship was represented by a table, a graph, and then a linear equation by the students. Students were also asked to identify the independent and dependent variables, and determine the domain and range. After a discussion on the definition of a relation, using a PowerPoint presentation to visually depict a relation, students were given four representations of a relation: braces, table, graph, and mapping. The PowerPoint presentation also had a nice illustration of a relation that is a function and a relation that is not a function. For example, a soda machine that gives one soda when purchased is a “functioning” relationship. If the soda machine gives more than one soda or none at all, it is not a “functioning” relationship. Another example described a relationship between a man and his girlfriend. If the man has one girlfriend, it is a “functioning” relationship. If he has more than one girlfriend, then it is not a “functioning” relationship. Once the students had a
picture of what it means to be a function, the four representations listed above were given again, but this time the students had to determine if they were functions or not. The class practiced identifying functions from many different representations and students were shown how to use the vertical line test for graphs.

Function notation was presented next by using well-known formulas and rewriting them in function notation to see the correspondence between the two. A “function machine” was also used to represent function notation so that students had a visual concept of what it means to evaluate a function. Students were asked to evaluate functions from an equation and a graph.

Now that all of the terminology of function had been introduced and students were familiar with the idea of function, it was time to start transformations of function. I introduced transformations of linear equations and general shapes (refer to Figure 5) hoping that once students understood how moving these figures affected the equation of the graph, that transformations of quadratics, square roots and absolute value would come much easier.

After this the students went to the computer lab. Once in the computer lab, each student worked at a computer on Geogebra lab. The goal of the lab was for students to use two sliders, labeled $m$ and $n$, to move the linear equation up, down, left, or right. I was hoping students would observe that each slider moved the line to result in the same transformation of the equation. For example: Moving slider $m$ to the left three units was the same as moving slider $n$ up three units which lead to the equation $y = 1x + 3$. The slope was not changed, only the $y$-intercept. The lab was completed in one fifty-minute class period. The next day, students were handed notes to be filled in based on the Geogebra lab they had completed the previous day. We discussed vertical and horizontal shifts as a class and practiced moving general shapes based on a description or equation (see figure 5).
In the following graph, \( y = f(x) \). Graph the translation \( y = 2 + f(x+3) \) on the same coordinate system.

\[ \begin{array}{c}
\begin{array}{c}
\text{Graph}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{Figure 5. Sample of basic transformation problem.}
\end{array}
\end{array} \]

To familiarize the students with the functions that they would be transforming, I used a graphing calculator activity that asked students to graph three parent functions: quadratic \((y = x^2)\), absolute value \((y = |x|)\) and square root \((y = \sqrt{x})\). As an introduction to these three functions, I wanted students to recognize the characteristics of each function and be able to identify the function from its graph. Using an x- and y-axis of -10 to 10, students graphed each of the equations on the graphing calculator and listed characteristics of each function. Students were also asked to identify the domain and range of each function. To sum up their findings, each student filled in a worksheet that identified each function by name, characteristics of the parent function (domain, range, x- and y-intercepts), and the graph of each function. After that, students were given transformations of the parent functions equations and told to use the graphing calculator to sketch a graph of each equation. They had to identify the domain and range of each graph. This activity was done during a fifty-minute class period and a lead-in to the Geogebra lab that was used the next day.
Before heading to the computer lab, I moved all of the desks in my classroom in a semi-circle around the Smart Board so that students could easily see my demonstration with Geogebra. I presented a short explanation of the tools and parameters that the students would need in order to complete the Geogebra lab. Once in the computer lab, the students were given the Geogebra activity that led them into the exploration of transformations of the quadratic, absolute value, and square root functions. The activity used sliders to move each of the three functions left, right, up, down and sliders to produce horizontal and vertical stretches. As students were moving the sliders, they had to answer questions that accompanied the activity. I composed questions with the purpose of guiding the students to observe how transformations affect the equation of the parent function while the function is moving according to the parameter chosen. Since the activity took place at the beginning of a ninety-minute block period, we were able to go back to the classroom and have a discussion on the findings of the lab. They made observations in their notes and problems were practiced on whiteboards at their desks.

As soon as the students had ample practice and a good understanding of transformations, I decided to go back to verbal representations of functions. My goal was to keep reinforcing the many ways that students could represent functions. I found an activity in an article written by Hartter (2009) that gave ten relationships in which students had to decide if the description was a function or not, and state the input and output variables of the function. If the relationship was not a function, students had to explain why not. Diagrams and graphs could also be used to help explain the description. One problem asks, “At the end of the course, David recorded the names of students who earned A’s and the names of students who earned B’s”, is this a function? Depending upon the interpretation and diagram, it could go either way.
Before moving on to the last two sections of the functions chapter, I wanted the class to review all of the concepts they had learned so far and take a short quiz over the information. Once the review and quiz were finished, we moved on to reflective properties of functions. As per previous Geogebra labs, I took the class to the computer lab to discover the behavior of the three parent functions we had been working with when a positive or negative number is included as a leading coefficient of the equation of the function (i.e., \( y = -3x^2 \)). The lab required students to enter leading coefficients on each parent function, answer questions, and write any observations made about the graph. There were two main ideas I was looking for, reflections over the x- and y-axes and maximizations and minimizations of the graph and how these graphs related to the function equations. As soon as we came back to the classroom, the students practiced graphing the three parent functions with stretches and shrinks and reflections over both axes.

The last piece of the functions unit was composition of functions. To introduce this topic, I found an activity on the NCTM website called “Successive Discounts”. This exercise had students work as groups of two to find the better deal; use a $5 off coupon then take 25% off a pair of jeans, or take 25% off then use a $5 off coupon. The activity involved using a graphing calculator to analyze the equations created for these two representations by determining when the discounts would come out to the same price. The problem then guided the students into creating composition functions from the original equations. Once the activity was completed, I guided the class through different composition problems including evaluating ordered pairs, numbers, and expressions.
Finally, it was time to take the post-test. The post-test was given during a ninety-minute block and completed by the entire class. Those students that did not finish came in the following day during a study hall period to complete the post-test.

**Documentation Instruments**

A separate notebook was kept for my field notes to be recorded. Each page in the journal was divided into three columns: descriptions, feelings, and interpretations. Each day that observations were made, the date, place, and setting were noted. Descriptions of students’ actions, reactions and quotes were noted in the first column. My feelings about the observations were reported in the second column. The third column was used to make inferences about the observations for each setting. This notebook was kept in a locked filing cabinet in my locked classroom every night.

<table>
<thead>
<tr>
<th>Date:</th>
<th>Place:</th>
<th>Setting:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descriptions –</td>
<td>Feelings –</td>
<td>Interpretations –</td>
</tr>
</tbody>
</table>

*Figure 6. Sample of Observation Journal*

While documenting students’ comments and thoughts as they were filling in their activity worksheets, I found it quite difficult to document everything that was being said. I used the library presentation area only once because students were spread out in a big space and it was hard to hear and see everything that was occurring. The computer lab was a much better space since students were bunched together making it easier to observe them in their work environment. Although the students were closer together in the computer lab, other classes would often be in the lab at the same time since my class was using only twenty-five of the fifty
computers available. Sometimes this would make it difficult to hear what my students were saying because of the loud noise level.

At the end of the school day, I tried to type in my observations, feelings and interpretations into a table like the one above as soon as possible. Even though I have never been a fan of journaling, I felt that keeping track of my observations helped me to identify where students’ misconceptions were. This, in turn, forced me to figure out how to address these misconceptions and change my teaching accordingly. As an example, in the Geogebra lab on transforming linear functions, many questions existed relevant to how to write a rule for the transformation of a linear equation. The question asked the following:

Students did not know if they should use \( y = mx + b \), words, or a rule involving the \( m \) and \( n \) sliders that were a part of the activity. When I was transferring my notes to my observation log, I started thinking about ways I could address this question so that the students knew exactly what I was trying to get them to recognize. Without the journaling, I did not know that I would have identified this particular problem.

Data Collection for Analysis

This study was performed to record how Geogebra, a DMS, could better help students visualize function transformations and their properties. Concerning the first research question, “In what ways will Geogebra change my students understanding of function?” data was collected and analyzed in several ways. Of the twenty-three students in the class, observations of the eighteen students in the study during Geogebra labs were documented in a table like the one listed above. This table was then entered in a Microsoft Word document. Observations were made on as many of the 18 students as possible, concentrating on comments about Geogebra and concentrating on how functions were being transformed. I was focused on any comments about
tools being used in Geogebra, such as the slider, and students’ remarks on the tools’ effects on the graphs of the functions.

The observation table was then divided into three recurring themes found during the study. From these themes, I was able to determine and write about how Geogebra changed my students’ understanding of function. The three predominant themes identified were: the operation of sliders, connections between Geogebra and the activity I created, and applying findings to the homework and assessments.

For the second question, “How does Geogebra help students visualize functions?” assignments were created to help students interpret their Geogebra findings to applications of different types of function problems. Activities were also used where students worked in pairs to make interpretations about function transformations. The final piece that was used to determine if Geogebra helped students visualize functions was the post-assessment. These answers, compared with the pre-assessment answers, gave information regarding the use of Geogebra in teaching function transformations.
Chapter 4

Findings

Objective

The purpose of this project was to determine if the DMS, Geogebra, would help students to better visualize and conceptualize functions and their transformations. Data was collected from a class of eighteen students in Algebra II using numerous Geogebra lab activities and classroom assignments. Also included in the data pool was information gathered from a pre-test given before the unit and a post-test given at the end of the unit. Since our school district uses district wide assessments for each unit, I used data from the district assessment that was given at the end of the functions unit. Finally, observations collected during the Geogebra lab activities were used with the filled in activities themselves to give insight into students’ thinking.

Assignments and Activities

During the functions unit, multiple Geogebra lab activities and classroom assignments were given for students to complete. Table 1 lists the assignments and activities in the order they were taught. The coursework can also be found in Appendix B.

Table 2

*Functions unit activities and assignments*

<table>
<thead>
<tr>
<th>Activity/Assignment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre Assessment</td>
<td>Pre-test given prior to unit</td>
</tr>
<tr>
<td>Geogebra Tutorial</td>
<td>Activities designed for students to practice using the tools in Geogebra.</td>
</tr>
<tr>
<td>15 Second Videos</td>
<td>Videos modeling different functions are shown. Students draw a graph to represent the video.</td>
</tr>
<tr>
<td>4.1 – Stories</td>
<td>Notes and assignment on creating graphs to match a story</td>
</tr>
</tbody>
</table>

*(table continues)*
<table>
<thead>
<tr>
<th>Activity/Assignment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBL Activity</td>
<td>CBL used to match graphs to data</td>
</tr>
<tr>
<td>Independent/Dependent</td>
<td>Identifying independent and dependent variables</td>
</tr>
<tr>
<td>Set 4.2 – What is a Function</td>
<td>Determining a function from ordered pairs, mapping, tables, and graphs</td>
</tr>
<tr>
<td>Set 4.2 – Domain and Range</td>
<td>Identifying a function from a vertical line test and finding domain and range from graphs</td>
</tr>
<tr>
<td>Function Notation</td>
<td>Using function notation to evaluate functions</td>
</tr>
<tr>
<td>Using Geogebra to Transform Linear Functions</td>
<td>Geogebra activity that explores transformations of linear functions</td>
</tr>
<tr>
<td>Transformation of Linear Functions</td>
<td>Notes and assignment for transforming linear functions and general functions</td>
</tr>
<tr>
<td>Graphing Calculator Activity to Introduce Parent Functions</td>
<td>Using a graphing calculator to identify characteristics, domain and range, and shapes of four parent functions</td>
</tr>
<tr>
<td>Family of Functions Sheet</td>
<td>Table summarizing findings from previous graphing calculator activity</td>
</tr>
<tr>
<td>Geogebra Transformation of Functions Lab</td>
<td>Geogebra activity to explore transformations of quadratic function, absolute value function, and square root function</td>
</tr>
<tr>
<td>Transformation of Functions Notes</td>
<td>Notes to summarize previous Geogebra activity and sample problems to practice</td>
</tr>
<tr>
<td>Quadratic Transformations</td>
<td>Assignment to practice quadratic function transformations</td>
</tr>
<tr>
<td>Graphing Transformations of 3 Parent Functions</td>
<td>Assignment to graph 3 parent functions</td>
</tr>
<tr>
<td>Is it a Function or Not?</td>
<td>Determine if 10 statements given describe a function or not</td>
</tr>
<tr>
<td>Stretches, Shrinks, and Reflections of Graphs Geogebra Activity</td>
<td>Geogebra activity to explore the coefficient, $a$, in each of the three parent functions and how it effects each function</td>
</tr>
<tr>
<td>Set 4.6 – Notes and Homework</td>
<td>Notes and assignment to practice graphing and writing equations with reflections and stretches and shrinks</td>
</tr>
<tr>
<td>Composition of Functions</td>
<td>Notes and assignment to practice evaluating composition of functions</td>
</tr>
</tbody>
</table>
Each of the assignments and activities were scored and recorded in a computer software grade book provided by the school district. The mean score for each assignment and activity is shown in Figure 7 while the completion rate for each is in Figure 8.

**Figure 7.** Mean score for each assignment in functions unit

**Figure 8.** Completion rate of each assignment in functions unit
Based on the data from Figure 7, the mean scores of the coursework ranged from 4.1 to 5 out of a possible 5. The lowest score was a 4.1 (82%) and the highest score was a 5 (100%). The assignments and activities were graded based on completion of all problems and accurate answers. Since most of the tasks had more than five questions, a proportion was used to find the score based on a five-point scale.

The completion rate of assignments shown in Figure 8 indicates a high level of completion. The lowest completion rate was 67% on the CBL activity. This was due to many students being absent the day of the activity and some did not return to complete the task. Besides this exercise, the rest of the assignments were completed with an 89% to 100% completion rate, with seven assignments having a 100% completion rate. Students seemed to understand the work and most of it was completed increasing the fidelity of the treatment.

**Pre- and Post-Assessment Data**

Given that there were only eighteen students completing the pre- and post-assessment, I chose not to include percent increase/decrease figures. Depending on the question, the amount of percent increase/decrease was not representative of the students’ answers. Instead, the questions from the assessments will be grouped according to what was being asked and figures will be given based on these smaller components.

The first set of problems asks the participant to use data from a figure and/or table and be able to graph this data and write a corresponding equation. Problem one in both pre-assessment versions and the post-assessment requires a table to be filled in based on a figure and then an equation is written from this information. All three parts of this question had an increase in accuracy (refer to figure 9). While parts a and b had an increase of three students answering correctly, part c had the most significant change. Only one student answered correctly.
on the pre-test. Five answered correctly on the post-test. It was interesting that three students used a recursive formula to represent the equation in the pre-test. While I did not consider this answer correct, I was surprised that students used this concept from the first semester. Another finding from the post-test showed that seven students were very close to the correct equation by having the correct rate of change but wrong starting value. This finding showed improvement in understanding from the pre-test.

![Correct answers to Question 1](image)

*Figure 9. Data from question 1 on pre-assessment*

Another question in the first grouping is problem two from versions A and B in the pre-assessment. In version A, students were asked to interpret a table and to answer questions based on this table and to also draw a graph with each axis labeled correctly. Part d had the most improvement from pre- to post- with two more students answering correctly (refer to figure 10). This piece of the question asked students to interpret the table and graph and to find the biggest jump in change of rate.
Figure 10. Question 2 on pre-assessment version A

In version B, the question is similar to version A, only students were also asked to find an equation to match the table and graph. Part c, writing an equation for the data, is of particular interest since only 6 out of 18 students answered correctly. Only one more person answered correctly after seeing the problem a second time from the pre-test. Refer to figure 11.

Figure 11. Question 2 on pre-assessment version B

The next question on both versions gives a story and four possible graphs to represent the story. Students were asked to pick the correct graph and explain their reasoning for each graph. The two versions each had a different story, but the post-assessment contained both stories. The data suggests that students were better able to interpret the graph for a Ferris wheel ride with
fourteen out of eighteen students answering correctly. Only six out of eighteen students were able to identify the correct graph for a woman climbing a hill and her speed as a function of time elapsed.

While the previous problems pertained to graphical, tabular, and verbal descriptions of function, the next two problems were primarily focused on function notation and the concept of infinity. Question four in both versions (figure 12) of the pre-assessment was question five in the post-assessment. The focus of this question was to ascertain if students could determine how many ordered pairs satisfied a linear inequality. While the accuracy of the graph of the linear inequality itself declined by two students, there was a two-student gain in identifying infinitely many solutions as the answer to the problem.

![Correct Answers to Question 4](image)

*Figure 12. Data from question 4 on pre-assessment*

On problem five there was significant improvement on parts *a* and *c* in that more students were able to interpret function notation correctly. On the other hand, only three people answered part *b* correctly. The difference between this question and the other two is that the ordered pair was not given for the point in question. Most students answered (2, 3) for part *b*, in fact, 10 students answered with this ordered pair. The three students that answered correctly noticed that
the scale on the x- and y- axes were not 1:1 but were in fact 2:6, giving the correct answer of (2,12). Data from questions four can be found in Figure 13.

![Correct Answers to Question 5](image)

**Figure 13.** Data from question 5 on pre-assessment

The final question of the assessment was a combination of three representations of a function; table, verbal, and graphical. The students had to match each verbal description of a function with its appropriate table and graph. The four functions were: (1) \( y = x^2 \); (2) \( y = x - 2 \); (3) \( y^2 = x \); and (4) \( xy = 2 \).
Version A of the pre-assessment had graphs A and B and version B had graphs C and D whereas the post-assessment included all four graphs. Instead of incorporating the data from all eighteen students for each graph, I chose to match the data from the post-test to the students who took each version. Students who completed version A of the pre-test were the only ones recorded in figure 14 for graphs A and B. Likewise, students who completed version B of the pre-test were the only ones recorded in figure 14 for graphs C and D.

The most noticeable results from graphs A and B was the drop in students being able to identify the correct table corresponding to $y = x^2$ and the correct rule for $y = x - 2$. On the other hand, there was an improvement in every category for graphs C and D by an average of two students.

**Figure 14.** Summary of problem 6 on pre- and post-assessment.
After matching each graph to its correct table, equation and rule, students then had to explain their reasoning behind the choices they made. Regardless of some of the students’ incorrect answers to matching graphs, tables, equations and rules, fourteen out of eighteen students were able to make at least one correct statement about the choices they made. Refer to figure 15 for pre- and post-assessment data regarding explanations for choosing as they did.

![Correct Answers to Explanations](image)

*Figure 15. Data for explanations*

**District Function Assessment**

Approximately once a year, a number of Algebra II teachers from the school district meet to discuss unit tests and make any changes that are deemed necessary. These tests are distributed to the remaining math teachers to be used throughout the following school year. The function unit assessment consists of the following types of problems:

1. Drawing a graph to match a story;
2. Domain/range and independent/dependent variables;
3. Identifying a function from ordered pairs, mapping, table, verbal description, and graph;
4. Transformation of a line;
5. Reflections of functions;
6. Transformations of quadratic, square root, and absolute value functions;
7. Composition of functions

Even though the eighteen students scored an average of 78% on the unit assessment, there were four kinds of questions that produced lower scores. The first type of problem posed
asked students to identify if an ordered pair, a mapping, a table, a verbal description and a graph were functions or not. Twelve out of eighteen students recognized ordered pairs and the verbal description correctly, eleven classified the mapping correctly, ten identified the table, and thirteen used the vertical line test on the graph appropriately.

Two problems on the assessment called for students to correctly reflect a function over the appropriate axis. On the first problem, ten out of the eighteen students were able to correctly reflect a function over the x-axis. Only nine were able to describe the correct reflection (refer to figure 16) on the second problem.

![Figure 16. Representation of a reflection over the x-axis](image)

Question 11 on the district function assessment requires students to graph a quadratic or absolute value function and then state the domain and range. Three of the 18 students identified both the domain and range correctly. Of the remaining 15 students, 11 answered the domain correctly or the range correctly, but not both. The last type of problem that had a poor performance involved students writing the equation of transformations of quadratic, absolute value, and square root functions. Eleven out of 18 students answered accurately, with four of the remaining seven missing only the vertical/horizontal stretch.
After examining the data from the district assessment for functions, I went back and looked at how individual students scored on this test compared to how they had been scoring on previous tests. Out of the eighteen students taking the test, three improved dramatically. Two of these students had a low C to low B average on the previous six assessments. The first student, student A, received a score of 93% on the functions unit assessment. The problems that student A answered incorrectly is the same problems that are listed above. When I asked student A why her score was so much higher on this test, she replied: “Everything just seemed to make sense. I was able to see how the graphs moved in my head and what the answer should be.”

The second student, student B, actually failed the function unit assessment. I wanted to find out why student B failed the test and asked her to meet with me after school so that we could go through the test. Student B admitted that she could not determine how to transform the functions and how the numbers for vertical and horizontal shifts affected the function. She also stated the domain and range did not make any sense to her and she was not sure how to determine if a relationship was a function or not. After working out a schedule, student B came in for three days to make corrections and use Geogebra to graph the functions that she had missed on the assessment. We also went through the function notes and Power Point presentation again to re-teach the function concept. Geogebra was then used to explore domain and range at depth. Once student B made all of the corrections to her test and felt she had learned how to transform functions using Geogebra, she was given a different version of the unit assessment. Student B missed only two problems on the retake, one on reflection and the other on domain and range. This was a huge improvement over her original test. When I asked student B what made the difference in the retake test, she replied: “Geogebra. I was able to see
how the graph moved and what it did to the equation, especially the slope.” By stating slope, student B explained later that this meant vertical and horizontal shift.

The final student, student C, had been receiving anywhere from an F to a low B on her district assessments. She missed the same problems described above as well as the shapes of the appropriate functions and some composition of function problems. Her grade on the functions unit was an 86%, her highest test score of the school year. Student C stated: “I learn best when we do computer stuff and things with the graphing calculator. I really liked all of the activities we did and I think all of them helped me to understand the test.” On the down side, two out of the 18 students received lower scores than on the previous tests.

After creating a pre- and post-assessment based on research findings, the district unit assessment on functions fell short on providing the information I was looking for. I found that there were more composition and linear transformation problems than needed and not enough transformation problems. There were only three problems each of writing the correct function equation from a transformed graph and drawing a graph from a given equation. I would also like the assessment to have more problems on identifying functions from different representations.

**Performance on Geogebra Activities**

Since the focus of this Plan B was to determine if DMS, such as Geogebra, aides students in the transformations of functions, I kept an observation log of any comments made while students were completing Geogebra activities. The first Geogebra lab required students to complete a tutorial to get familiar with the tools available. To accommodate the twenty-four students in the class, we went to the library presentation area during a 90-minute block day. On the south side of the library is a smart board connected to a projector for teachers to attach their computer. Students are arranged around six tables, with four chairs each in front of the smart
board and another five desktops along the eastern wall. Students sitting at tables use laptops available from the librarian.

While the students’ computers were booting up, I handed out the tutorial that the students would be using. Once everyone was ready to begin, I started at the beginning of the tutorial and went through the first three pages together. I then let the students work at their own pace to complete the rest of the tutorial on their own. As students were working on the activity, I walked around to each table and desktop area to answer any questions and record comments. While most students worked hard to complete the tutorial, several were off task and drawing various pictures. My observations and recorded comments revealed that students were trying different tools to create various graphs and pictures and they would only occasionally get stumped. Some comments I heard were: “How do I get the toolbar?”; “How do I get the grid?”; “How do I undo?”; “I found an eraser!”; “How do we enter function?”; “Is the input where I enter it?”

There was a lot of cooperative learning among the students and I felt that students grasped how to use the tools in Geogebra to achieve the tutorial’s purpose.

The next lab was completed in the computer lab during a 90 minute block day where the first 50 minutes was spent doing the Geogebra lab and the last 40 minutes was a wrap up of the findings. The activity on this day used Geogebra to transform linear functions. The computer lab is located on the west side of the high school with 50 computers arranged in rows comprised of four computers to one table and two computers on another table at the end of the row. My class was seated at the rear of the computer lab using a block of 24 computers.

The students went through the lab answering the questions that were posed to them. There were multiple questions during this activity as I was constantly walking back and forth between the students and their computers. The first part of the Geogebra activity asked students
to move a slider and describe the transformation. While some students identified the slider they used, most described the transformation without identifying the slider. The consensus among the eighteen students involved in the study was that the m slider moved the linear function to the right or left. They also phrased this as the line moving to the right or left, not up or down; the x-coordinate moved right or left; the x shifted on the x-axis; and if you add a number to the x-coordinate it tells how many units to shift to the right or left.

The second part of the activity was the same, except that vertical transformations were the subject of interest. Again, some students identified the n slider moving the linear function up or down. For the most part though, students used such phrases as the line is moving vertically, not horizontally; points went up the y-axis; add to the y-coordinate tells how many units to shift; and to go up use n to change the y-axis by one.

During the time in the computer lab, almost the entire class had questions about the third part of the activity. This section asked students to find a rule to represent vertical transformations and a rule to represent horizontal representations. I did not want to impose my thinking on the students so they were told to answer the question so that it made sense to them and the person next to them. Whereas I was hoping students would define the rule in terms of the m and n sliders, this was only the case for two of the 18 students. The rest used words to describe the rules for transformations. I was also expecting the class to see the connection between the sliders: if slider m moves right, it is the same as slider n moving down and if slider m moves left, it is the same as slider n moving up (refer to Figure 14). This was not the case. After the lab, we went back to the class to wrap up our findings. There was a lot of discussion when we got back to the classroom about the connections between the two sliders and what exactly it meant to transform a linear function.
After the linear function portion of the unit, we went on to transformations of the quadratic, absolute value, and square root functions. As an introduction, the class gathered in a semi-circle around my smart board and I used my computer and projector to show how to explore parameters of a quadratic polynomial in Geogebra. This was done during the first 15 minutes of the 90-minute block day. Then the class went to the computer lab for the next 45 minutes to complete the Geogebra activity I had created on transformations of functions using sliders. I was amazed at how well this activity went. Every student was genuinely involved in the activity and wrote down answers to each question asked. One example is based on the first section of the transformations and that is how the slider $a$, which is the leading coefficient, affects the shape of the function. One student wrote, “$a$ changes the relationship between $y$ and $x$, making the difference larger or smaller and therefore changing the curve of the graph.”

*Figure 17. Sample of sliders $m$ and $n$ from linear transformation activity*
As students progressed through the activity, they were helping each other instead of asking me questions and not making many comments because they were engrossed in the activity. Another significant comment occurred during the transformations that used a slider to move the function right and left. A student stated, “$h$ positive numbers create a subtraction in parenthesis and moves left where negative numbers create addition in parenthesis and move right”. This student was referring to the $h$ slider. The only real problem I found with this activity was how to change from a quadratic to absolute value function and then absolute value to square root function. The slider for this change in function was located in the lower left of the screen and not labeled large enough to notice.

The final activity used Geogebra to explore the coefficient $a$ and how it affected graphs of quadratic, absolute value, and square root functions. This was also another great activity for the reason that students were once again involved and helping each other to answer the questions. There were not many comments being made but the questions in the activity were answered completely and with some thought. For the question “Write a statement describing how a coefficient in front of x affects the graph” a student answered with the following: “The coefficient controls the direction and vertex placement”. I asked the student what he meant by vertex placement and he elaborated that he thought the coefficient affected the size of the function.

During the course of the observations made during Geogebra labs, the completed activities and course work, the pre- and post-assessment, and the district assessment on the functions unit, it became clear that all of this data pointed to some interesting revelations about the transformations of functions and how students interpreted the these transformations. The next chapter will reveal these findings and their implications.
Chapter 5

Conclusions

While reading research articles for the literature review, it became apparent that many factors affect students’ learning of functions and their transformations. Even though Geogebra had an impact on students’ visualization of the transformations of functions, some predominant misconceptions occurred. The two prevailing misunderstandings were the transformations of linear functions and using procedural instead of conceptual understandings when transforming quadratic, absolute value and square root functions.

Literature Review and Implications

According to a case study done by Chiu et al. (2001), and Munoz-Nunez (2001), students tend to see line segments moving as horizontal translations instead of vertical. They define a conception as a set of coherent and connected ideas associated with a specific strategy or with several related strategies. From past research, understanding linear functions involves the connections between representations. The vertical translation strategy tends to be more straightforward when focusing on the equation and the y-axis (Chiu, et al., 2001). Their study shows that when students first learn lines, they tend to view parallel lines as horizontal instead of vertical translations of each other. These two conceptions show the importance of allowing for multiple strategies in the classroom.

After students in my Algebra II class participated in a Geogebra activity aimed at connecting a linear transformation to its equation was completed, I found that most students had difficulty making this connection. During the activity, the majority of the students were unsure of how to write a rule for a linear transformation. Only two students recognized the link between
the two sliders in the Geogebra lab. That is, if the m slider was moved to the right, which moved the line to the right, this was the same as moving the n slider down the same amount of units (refer to Figure 16). Eventually, students tended to memorize the “rule” instead of truly understanding; it became procedural instead of conceptual.

Once the lab was completed, the students came back to the classroom and we discussed the activity. It became obvious that the students did not really understand what connections the activity was trying to get them to discover. As an afterthought, I should have asked students to create the same line using the two sliders individually. Although the students seemed to comprehend our discussion and the implications on linear transformations at the time, the quiz and test indicated otherwise. Using tables to illustrate a linear transformation might have helped with the visualization piece as shown in Figure 18.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>-2</td>
</tr>
</tbody>
</table>

Transform the line right 2 units and up 5 units.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Resulting equation: \( y = \frac{-2}{3} x + 8 \frac{1}{3} \)

Figure 18. Transformation of linear function with tables

Another approach could have been to graph linear functions from their equation. As soon as students had an understanding of slope and y-intercept, transformations could then be graphed. A comparison of the original linear function and its transformation could then lead to the effect of the transformation on the equation of the linear function. The Geogebra lab was
meant to help with the visualization of a linear transformation, but instead caused confusion. I believe using tables and graphs could have increased the students’ understanding of linear transformations.

In past years, linear transformations were barely touched upon, if at all. This year was the first time that I had taught linear transformations in detail. My thinking was to use linear transformations as a stepping-stone to transformations of general figures (not necessarily quadratic, absolute value, or square root functions). After teaching this unit, I believe it would be better to start with transformations of quadratic, absolute value, and square root functions and then transform some general figures. I am not so sure if linear transformations should be addressed in this unit.

Even though linear transformations did not progress as I had hoped, quadratic transformations on the other hand, went above my expectations. Students were able to visualize the transformations left, right, up and down after doing the Geogebra activity. Nearly ninety percent of the students scored every transformation problem correct on the district assessment. These problems involved graphing a transformation from a function equation and writing a function equation from a graph. The functions used were quadratic, square root, and absolute value and the transformations were left, right, up and down. In previous years, many students seemed to mix up the equation for moving right or left. They memorized the procedure instead of truly “seeing” the function move.

Throughout the literature review, a recurring theme emerged, i.e., DMS and graphing calculators help students to better visualize transformations of functions. It was also claimed repeatedly through the literature review that many representations of function should be taught and used for students to have a conceptual understanding of what it is that defines a function.
From my results in chapter four, it was clear that I did not do enough with the different representations of function. In the beginning of the unit, the class worked almost exclusively on defining and identifying functions in its various forms. As we began to evaluate and transform functions, we worked more with the equations and graphs of functions and less on mapping, ordered pairs, and tables.

After attending a Geogebra conference in Ithaca, NY, I designed a Geogebra program that allows the user to move various sliders affecting the graphs in various ways. One slider changed the graph between quadratic, square root, and absolute value functions. The other three sliders move the function left/right, up/down, or affect the width and direction of the function. Aside from a few minor adjustments that need to be made to the program, every student was completely involved in the designed activity that accompanied the program and answered each question correctly. The data from chapter four shows that graphing transformations and writing equations from transformations had high scores.

On the other hand, reflections and horizontal and vertical stretches were not as successful as the transformations. One of the reasons may be that I used my own Geogebra activity, recently created for the functions unit, which did not ask very many follow up questions about the effect of a coefficient other than positive one. The Geogebra lab visually represented reflections, but horizontal and vertical stretches were not incorporated. These were addressed later in the classroom with students completing tables from an equation and then drawing the graph by hand.

Limitations

Since most of my research was a result of using technology, whether it was Geogebra on a computer, graphing calculators, or CBL’s, many glitches occurred. There were times that the
computer was either very slow, or would not let the students run Geogebra because they needed the current version of Java. Some of the CBL’s that were used were fairly old and would stop working for no apparent reason. After the first occurrence of technology malfunction, I made sure to have other activities available to use.

While writing down observations of students’ comments, I would sometimes find myself defending the Geogebra software to students who did not like using it or were having problems using the software. It was hard for me to not interject my feelings about a tool that I have found valuable and of importance to me. I changed my observation journal from two columns to three by adding the column labeled “feelings”. This allowed me to journal my feelings about the activity and any comments the students made.

Even though I did gain a lot of information about using Geogebra to help teach function transformation, I feel I could have learned more had I also used a control group. Because I only had one Algebra II class during the research process, a control group was not an option. Another limitation was time. As with most schools, I have a certain amount of curriculum I have to teach before the end of the school year. Therefore, I was not able to spend as much time as I had wished on the functions unit.

One last limitation had to do with the pre- and post-assessment and the IRB. To get the IRB turned in with enough time to conduct my study, I had to create the pre- and post-assessments without completing all of my research for the literature review. I feel that if I had time to complete the literature review before creating the assessments, I would have generated problems that addressed the obstacles behind the learning of the function concept.
Why Now?

Ever since I started teaching this unit seven years ago, I have felt that it was one of the hardest units to teach and the hardest for students to conceptually understand. During the first few years, I would borrow lesson ideas and homework from colleagues and tweak them to fit my needs. While the suggestions and ideas from others helped, I still thought that something was missing. Students were not performing as well as I had hoped on the district assessment and I took this as a direct reflection of my teaching.

Two years ago after my oldest son turned 12 and was able to stay home with his younger brothers; I enrolled in my first class through the Science and Mathematics Teaching Center. A friend and colleague had just finished the program and always had great ideas to use in the classroom. I wanted to be exposed to these same concepts and to become a better teacher and joining the program seemed the next step. Once it came time to choose a topic and question for my Plan B paper, the functions unit immediately popped into my mind.

With so many choices of technology available to use in the classroom, it seemed appropriate to see how one would help or hinder students’ understanding of function. Our school has iPads available in the library, lap top and desktop computers in the library and computer lab, and graphing calculators, document cameras and Smart boards in the classroom. It seems that I am usually looking to the Internet for new and innovative ways to teach the various concepts seen in my classroom. One of the main websites I use is the Geogebra site. I have been using the function transformation activities for the past three years in the functions unit and have been pleased with the results. I also enjoy using technology in the classroom and my students seem to enjoy it as well. Since I am on the computer during planning trying to find the best way possible
to teach the next concept at hand, the logical next step for me was to incorporate GeoGebra throughout as much of the functions unit as possible and see what the data would tell me.

**What is Next?**

During the function unit, I not only learned much about my students and their understanding of function, I learned considerably about myself as an educator. Prior to this research paper, I had been mainly interested in covering all of the material and having students work a large amount of practice problems. I also did not pay much attention to whether students understood the many facets of the function concept and if they were making connections among the different representations of function. The research procedure has changed my instruction and homework activities. Instead of just lecture and note-taking, I now look for thought-revealing activities that students can do on their own or with a partner so that any concept taught becomes conceptualized instead of procedural.

I had been previously trained in how to ask the right questions so that I could understand their thinking process. The research process of this paper strengthened this skill and helped me to listen closely to what students were saying. During the GeoGebra activities, I was documenting in my observation journal what students were saying and doing. With this close-up interaction, I caught myself continually asking students to explain their statements, which led to deeper understanding on my part. Because of this interaction, I feel that the students and I feel free to question each other leading to an open environment in my classroom. This knowledge that I have gained from the research process had allowed me to gain information about students’ thinking that I might not have considered before.

An example of the change in my classroom, as a result of this Plan B project, occurred in my Algebra I class. This is a lower level group of students comprised mostly of sophomores. As
an introduction to the function unit, I found a lesson that used secret codes. From a graduate class I had taken through SMTC, I showed the students a series of 10-minute videos telling the tale of Bletchley Park cryptographers during WW II. These videos grabbed the students’ attention and they immediately wanted to know what this had to do with math. The class then completed a series of secret messages using a basic transposition code (the letter A would be moved a certain amount of times to create a code). During this time, a code was introduced that had two possible letter outputs for certain inputs. This was a great lead in to what determines a function. I then found a website that allows the user to print off a Caesar’s Cipher and a Mexican Army Cipher. The class cut these out, assembled the ciphers, and sent secret messages to each other. When it came time to go over the different representations of a function, the class was hooked.

**Future Research**

One of my hopes is that another math educator will decide to do a research on the best way to teach functions and their transformations to help other teachers, like me, improve. From my study, I believe the Geogebra transformations activity was a great asset. The other two activities, linear transformations and reflections, could definitely use some refining. Although the literature review had many researchers stating that functions were best learned by working with the many representations of function, I am not sure that I found this to be true in my study. My class did different activities that involved learning and using all of the different representations to identify and solve functions. However, when it came to the district assessment and the pre- and post- assessments, this was the weakest area of the unit. If a future researcher could address and find the best method for introducing functions and their representations so that
it became completely conceptual, I think this would greatly improve the rest of the functions unit.

Ultimately, this research paper has taught me that I am ever changing as a teacher and as a student of learning. There are many ways that I can improve what I am doing in the classroom and the functions unit is just the beginning. As long as I am willing to repeatedly question what I am doing and why, and to see if there is anything else that could be done to become a better teacher, I will continue grow as a person and educator alike.
References


APPENDIX A

PRE- AND POST-ASSESSMENTS

PRE-ASSESSMENT: FUNCTIONS

VERSION A

Name:________________________

Please answer each question completely.

1. Suppose girder beams are made with short steel pieces like the one drawn here:

For example, below is a beam we call a “6-beam”: its longest edge has the length of 6 of short steel pieces. Can you explain why “6-beam” is a good descriptor for this beam?

a) How many short steel pieces are used to make a 6-beam?

b) Fill in the chart to show the number of short steel pieces used in girders with various lengths.

<table>
<thead>
<tr>
<th>Length</th>
<th># Pieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

2) Find a pattern in the chart. Try to make a rule for any length, n.

2. The following table shows some data Carmen collected during her swim team practice.

<table>
<thead>
<tr>
<th>Number of breaths</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of meters swum</td>
<td>0</td>
<td>5</td>
<td>8</td>
<td>12</td>
<td>15</td>
<td>17</td>
<td>20</td>
<td>24</td>
</tr>
</tbody>
</table>

a) What are the two variables?________________________

b) Graph the data from the table on the axes to the right.
   Remember to label the axes and write a title.

   c) Does it make sense to connect the points? _______
      Explain your reasoning:
d) When did Carmen make the most progress? How does this show up in the table?

How does this show up in the graph?

e) How many breaths do you think she would take if she swam 50 meters? Explain why:

3. Explain which graph best represents a woman climbing a hill at a steady pace who, reaching the top, starts to run down that same hill. Make sure to include all of your reasons for choosing as you did.

4. In the coordinate graph to the right, graph every point that satisfies the inequality: \( y < x + 1 \)

How many points satisfy the inequality?
5. Using the graph to the right, determine each of the following function values:

a) \( f(-2) \) 

b) \( f(2) \) 

c) \( f(8) \) 

6. At the bottom of the page are two graphs, two equations, two tables, and two rules. Your task is to match each graph with an equation, a table and a rule. Record your answers in the table below, and, on under the table, explain how you matched each of the two graphs to its equation.

   a) Write your answers in the following table.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
<th>Table</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( y = x - 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>( y = x^2 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b) Explanation:
PRE-ASSESSMENT: FUNCTIONS

VERSION B

Name: ________________________________

Please answer each question completely.

1. Suppose girder beams are made with short steel pieces like the one drawn here:

For example, below is a beam we call a “6-beam”: its longest edge has the length of 6 of short steel pieces. Can you explain why “6-beam” is a good descriptor for this beam?

a) How many short steel pieces are used to make a 6-beam?

b) Study the girder above. Fill in the chart using this way of counting:

\[(\# \text{ on bottom}) + (\# \text{ on top}) + (\# \text{ slanting})\]

<table>
<thead>
<tr>
<th>Length</th>
<th># Pieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

c) Using this way of counting, what would the rule be for a length, \(n\)?

2. Imagine you are in your hotel on vacation in Southeast Asia. You feel you are coming down with a severe fever. How would you find your body temperature? Looking around, you find a fever thermometer that is marked in degrees Celsius. The thermometer registers your temperature as 40° C. Should you be concerned? You understand the meaning of Fahrenheit readings of body temperature, but don’t know how to interpret the Celsius readings. Then you find a partial chart like the one below, showing some Fahrenheit measurements and the corresponding degrees in Celsius.

<table>
<thead>
<tr>
<th>(F^\circ)</th>
<th>(C^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>212</td>
<td>100</td>
</tr>
<tr>
<td>158</td>
<td>70</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>-4</td>
<td>-20</td>
</tr>
</tbody>
</table>
a) Using the table, make a graph comparing F to C.

b) Write a statement about the relationship between F and C, describing as much as you can about how the scales are related.

c) Represent the relationship between F and C with an equation.

3. Explain which graph best represents a man taking a ride on a ferris wheel. Make sure to include all of your reasons for choosing as you did.

4. In the coordinate graph to the right, graph every point that satisfies the inequality: y < x + 1

How many points satisfy the inequality?
5. Using the graph to the right, determine each of the following function values:
   a) \( f(-2) \) __________
   b) \( f(2) \) __________
   c) \( f(8) \) __________

6. At the bottom of the page are two graphs, two equations, two tables, and two rules. Your task is to match each graph with an equation, a table and a rule. Record your answers in the table below, and the table, explain how you matched each of the two graphs to its equation.
   a) Write your answers in the following table.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
<th>Table</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b) Explain how you matched each of the four graphs to its equation.

   Graph C:

   Graph D:
POST-ASSESSMENT: FUNCTIONS

Name: ________________________________________________
Please answer each question completely.

1. Suppose girder beams are made with short steel pieces like the one drawn here:
   For example, below is a beam we call a “6-beam”: its longest edge has the length of 6 of short
   Steel pieces. Can you explain why “6-beam” is a good descriptor for this beam?

   a) How many short steel pieces are used to make a 6-beam?

   b) Fill in the chart to show the number of short steel pieces used in girders with various lengths.

   c) Find a pattern in the chart. Try to make a rule for any length, n.

   d) Study the girder above. Fill in the chart using this way of counting:

      (# on bottom) + (# on top) + (# slanting)

   e) Using this way of counting, what would the rule be for a length, n?
2. The following table shows some data Carmen collected during her swim team practice.

<table>
<thead>
<tr>
<th>Number of breaths</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of meters swum</td>
<td>0</td>
<td>5</td>
<td>8</td>
<td>12</td>
<td>15</td>
<td>17</td>
<td>20</td>
<td>24</td>
</tr>
</tbody>
</table>

a) What are the two variables? ____________________________

b) Graph the data from the table on the axes below. Remember to label the axes and write a title.

![Graph](image)

c) Does it make sense to connect the points? _____
   Explain your reasoning:

![Graph](image)

d) When did Carmen make the most progress?
   ____________________________
   How does this show up in the table?

   ____________
   How does this show up in the graph?

   ____________

e) How many breaths do you think she would take if she swam 50 meters?
   Explain why:

3. Imagine you are in your hotel on vacation in Southeast Asia. You feel you are coming down with a severe fever. How would you find your body temperature? Looking around, you find a fever thermometer that is marked in degrees Celsius. The thermometer registers your temperature as 40°C. Should you be concerned? You understand the meaning of Fahrenheit readings of body temperature, but don't know how to interpret the Celsius readings. Then you find a partial chart like the one to the right, showing some Fahrenheit measurements and the corresponding degrees in Celsius.

<table>
<thead>
<tr>
<th>F°</th>
<th>C°</th>
</tr>
</thead>
<tbody>
<tr>
<td>212</td>
<td>100</td>
</tr>
<tr>
<td>158</td>
<td>70</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>-4</td>
<td>-20</td>
</tr>
</tbody>
</table>
a) Using the table, make a graph comparing F to C

b) Write a statement about the relationship between F and C, describing as much as you can about how the scales are related.

c) Represent the relationship between F and C with an equation.

4. In the following two situations, explain which graph best represents the situation. Make sure to include all of your reasons for choosing as you did.

a) A man takes a ride on a ferris wheel.

b) A woman climbs a hill at a steady pace and then starts to run down one side.
5. In the coordinate graph to the right, graph every point that satisfies the inequality: \( y < x + 1 \)

How many points satisfy the inequality?

6. Using the graph to the right, determine each of the following function values:

a) \( f(-2) \) __________

b) \( f(2) \) __________

c) \( f(8) \) __________
7. On the next page are four graphs, four equations, four tables, and four rules. Your task is to match each graph with an equation, a table and a rule.

a) Write your answers in the following table.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
<th>Table</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Explain how you matched each of the four graphs to its equation.

Graph A:

Graph B:

Graph C:

Graph D:
<table>
<thead>
<tr>
<th>Graph A</th>
<th>Equation A</th>
<th>Table A</th>
<th>Rule A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$xy = 2$</td>
<td>$x$ (-2) (-1) 0 1 2 3 $</td>
<td>$y$ (-4) (-3) (-2) (-1) 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y$ is the same as $x$ multiplied by $x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graph B</th>
<th>Equation B</th>
<th>Table B</th>
<th>Rule B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^2 = x$</td>
<td>$x$ (-2) (-1) 0 1 2 3 $</td>
<td>$y$ 4 1 0 1 4 9</td>
<td>$x$ multiplied by $y$ is equal to 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graph C</th>
<th>Equation C</th>
<th>Table C</th>
<th>Rule C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2$</td>
<td>$x$ 0 1 4 9 16</td>
<td>$y$ 0 $\pm1$ $\pm2$ $\pm3$ $\pm4$</td>
<td>$y$ is 2 less than $x$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graph D</th>
<th>Equation D</th>
<th>Table D</th>
<th>Rule D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x - 2$</td>
<td>$x$ (-2) (-1) 0 1 2 4</td>
<td>$y$ (-1) (-2\ $\pm\infty$ 2 1 0.5</td>
<td>$x$ is the same as $y$ multiplied by $y$</td>
</tr>
</tbody>
</table>